LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2024**

UMT 5502 – LINEAR ALGEBRA

Date: 11-11-2024 Dept. No. Max. : 100 Marks

Time: 09:00 am-12:00 pm

SECTION A

Answer ANY FOUR of the following **4 x 10 = 40 Marks**

1. If V is a vector space over F, then prove the following:
   (i) $\alpha .0=0, for\, all\, \alpha \in F$
   (ii) $0.v=0, for\, all\, v \in V$
   (iii) $(-\alpha)v=-(\alpha v), for\, all\, \alpha \in F, v \in V$
   (iv) if $v \neq 0$, then $\alpha v=0$ implies $\alpha=0$.

2. State and prove Schwarz inequality.

3. If V is of finite dimensional vector space over F, then show that $T \in A(V)$ is singular, if and only if there exists a non-zero vector $v \in V$ such that $vT=0$.

4. If V is of finite dimensional vector space over F, then $S, T \in A(V)$, then prove the following:
   (i) $r(ST) \leq r(T)$
   (ii) $r(TS) \leq r(T)$

5. Prove that if $T \in A(V)$ is unitary if and only if $TT^{¿}=1$.

6. Let F be a field and let V be the set of all polynomials in x of degree n-1 or less over F. On V let D be defined by $(\beta_0+\beta_1 x+\ldots+\beta_{n-1} x^{n-1})D=\beta_1+2\beta_2 x+\ldots+i\beta_i x^{i-1}+\ldots+(n-1)\beta_{n-1} x^{n-2}$. It is trivial that D is a linear transformation of V. What is the matrix of D?

7. If N is normal and $AN=NA$, then show that $AN^{¿}=N^{¿}A$.

8. If $W \subset V$ is invariant under T, then show that T induces a linear transformation $\overline{T}$ on $\frac{V}{W}$,defined by $(v+W)\overline{T}=vT+W$. Further, If T satisfies the polynomial $q(x) \in F(x)$, then so does $\overline{T}$. And if $p_1(x)$ is the minimal polynomial for $\overline{T}$ over F and if $p(x)$ is that for T, show that $p_1(x) \vee p(x)$.

SECTION B

Answer ANY THREE of the following **3 x 20 = 60 Marks**

9. If V is the internal direct sum of $U_1, U_2, \ldots, U_n$ then show that V is isomorphic to the external direct sum of $U_1, U_2, \ldots, U_n$.

10. State and prove Gram Schmidt orthogonalization process.

11. Prove that $Hom(V,V)$ is an algebra over F.

12. If $(vT, vT)=(v,v)$ for all $v \in V$, then show that T is unitary.

13. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

14. “The linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V” – Justify.